

Cosmic (super)string constraints from 21 cm radiation

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We calculate the contribution of cosmic strings arising from a phase transition in the early universe, or cosmic superstrings arising from brane inflation, to the cosmic 21 cm power spectrum at redshifts $z \geq 30$. Future experiments can exploit this effect to constrain the cosmic string tension $G\mu$ and probe virtually the entire brane inflation model space allowed by current observations. Although current experiments with a collecting area of $\sim 1 \text{ km}^2$ will not provide any useful constraints, future experiments with a collecting area of $10^4 - 10^6 \text{ km}^2$ covering the cleanest 10% of the sky can in principle constrain cosmic strings with tension $G\mu \gtrsim 10^{-10} - 10^{-12}$ (superstring/phase transition mass scale $> 10^{13} \text{ GeV}$).

Introduction. — There has been a revival of interest in cosmic strings, the line like topological defects of cosmic length, after it was found that they can arise in the superstring theories in braneworld inflation scenarios [1]. They are also found to be inevitable in a wide class of supersymmetric Grand Unified Theories (GUTs) [2]. Historically cosmic strings were found to form in GUTs during phase transitions in the early Universe along with the other topological defects like monopoles and domain walls [3]. Unlike monopoles and domain walls, which very quickly dominate the energy density of the Universe if formed after inflation, strings approach a scaling solution and can remain sub-dominant [4]. In brane inflation only cosmic strings and no monopoles or domain walls are produced [5].

Cosmic strings were proposed as a mechanism for generating the primordial fluctuations which later grew to form the large scale structures we see today [6]. The Cosmic Microwave Background (CMB) anisotropies and the matter power spectrum arising from the fluctuations seeded by cosmic strings are very different from those generated from inflation [7]. Inflation just prescribes the initial fluctuations at the end of inflation generated once and for all, which then just evolve. Cosmic strings generate fluctuations throughout the history of the Universe. One effect of this is that the fluctuations generated at different times add up out of phase and wash out the acoustic oscillations in the CMB. The discovery of the acoustic peaks by CMB experiments [8, 9] was a major success for inflation and ruled out cosmic strings as the dominant mechanism for seeding the cosmic perturbations. A sub-dominant contribution from cosmic strings to the cosmic perturbations is still not ruled out with the current constraint being $G\mu \lesssim 10^{-7}$ for classical strings [10]. Cosmic strings if discovered either through their gravitational lensing effects [11], through the gravitational waves

produced at string cusps or decaying loops [12] or through their effect on the CMB and the matter power spectrum will provide insight into the fundamental physics at high energies which is beyond the reach of currently planned terrestrial experiments.

21 cm radiation from $z \geq 30$ is an excellent probe of the state of the Universe at that time. This radiation can probe much smaller scales than the CMB, in the redshift range $30 \leq z \leq 200$, and provides a three dimensional view of the Universe before reionization [13]. It has been shown to be an excellent probe of the fundamental physics like variation of constants (fractional variation in the fine structure constant of $\lesssim 10^{-5}$ with 10^4 km^2 collecting area), non-Gaussianity from inflation (non-Gaussianity parameter $f_{nl} \sim 0.01 - 1$), dark matter and inflationary fundamental physics [14, 15, 16]. In this *Letter* we show that it can, in principle, put unprecedented tight constraints on the cosmic string contribution to the perturbations in the matter or equivalently on the string tension $G\mu$, and possibly other parameters, which translates into constraints on the GUTs and the superstring theory. G is the gravitational constant and μ is the string mass per unit length so that $G\mu$ is dimensionless.

Cosmic strings. — Cosmic strings arise in GUTs and superstring theories whenever there is a phase transition in the Universe if the vacuum manifold contains unshrinkable loops, e.g. $U(1)$. The superstring theory can produce a variety of cosmic strings, which can be fundamental (F-)strings or D-strings produced during annihilation of D-branes. The string tension for these strings in brane inflation models is $10^{-12} \lesssim G\mu \lesssim 10^{-6}$ [5, 17, 18]. Just like the classical cosmic strings from GUTs, they are wiggly, can intercommute and form loops which can decay into gravitational radiation or elementary particles. The main difference from the classical cosmic strings of GUTs is that their intercommuting probability can be less than unity. Also different kinds of strings can form bound states [19]. The superstring theory string networks have scaling solutions [20] just like the classical strings [4] i.e. the total length of the strings inside a horizon volume

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is proportional to the horizon size at any time and the string energy density is a constant fraction of the dominant energy density component of the Universe in the radiation dominated as well as matter dominated eras, preventing them from dominating. This also makes possible to construct simpler models of string networks which can than be used to study their impact on cosmology. Also since the perturbations produced by cosmic strings are independent of the inflationary initial conditions, the two kind of perturbations can be evolved independently and the resulting power spectra for CMB or matter added together to get the total power spectrum.

The long wiggly cosmic strings have a structure that resembles a random walk on scales larger than the horizon but are straight on small scales [21]. We use the CMBACT code developed by Pogosian and Vachaspati [22] which is based on CMBFAST [23]. Wiggly strings are modeled in the code as independent pieces of small strings whose length is taken to be of the order of the correlation length of the pieces of strings derived from numerical simulations. The intercommuting probability (P) is taken to be unity. The wiggleness on small scales results in the effective string tension and mass per unit length of the string observed by a distant observer to differ [24] with the equation of state $\tilde{U}\tilde{T} = \mu^2$, where \tilde{U} and \tilde{T} are the effective mass per unit length and tension of the wiggly string. Thus at scales greater than the scale of wiggles, the matter around experiences a Newtonian gravitational potential in addition to the deflection due to the conical space around the string [25]. The wiggleness can be controlled in CMBACT by a wiggleness factor α defined by the equations $\tilde{U} = \alpha\mu$ and $\tilde{T} = \mu/\alpha$ [22].

21 cm radiation– After recombination most of the baryonic matter is in the form of neutral atomic hydrogen and helium with a very small residual ionization. This small fraction of electrons couples the CMB to the matter up to $z \sim 500$ through Compton scattering and maintains the gas at the same temperature as the CMB. Around $z \sim 500$ the Compton scattering timescale becomes larger than the Hubble time and the gas decouples from the CMB and cools adiabatically thereafter. The ground state of hydrogen atom has a hyperfine splitting with an energy difference of $T_\star = 0.068$ K. This corresponds to the $\nu_{21} \sim 1420$ MHz rest frequency or $\lambda_{21} \sim 21$ cm rest wavelength. The hydrogen gas will absorb or emit photons of this energy depending on the population levels in the two states which is best parameterized by the spin temperature T_s defined by the equation $n_t/n_s = g_t/g_s e^{-T_\star/T_s}$. Here n_t and n_s are the number densities of hydrogen atoms in the excited triplet state and the ground singlet state respectively, $g_t = 3$ and $g_s = 1$ are the corresponding statistical weights. The population levels during $z \geq 30$ can change through collisions or through emission and absorption of CMB photons. Initially the collisions dominate over the radiative process and T_s follows the gas temperature T_g . At late times the density of gas becomes too low for collisions to be effective and T_s approaches the CMB temperature,

$T_\gamma = 2.725(1+z)$. Thus in the redshift range of about $500 \geq z \geq 30$, $T_s < T_\gamma$ and we have a net absorption of the CMB [13, 26]. At $z \sim 30$ the first stars are born and the evolution of the Universe enters non-linear regime. We will focus on the redshift range $200 \geq z \geq 30$ in this *Letter*, which corresponds to the observed frequency of $\sim 7.1\text{MHz} \leq \nu \leq 47.3\text{MHz}$.

The observed intensity I_ν can be expressed in terms of the brightness temperature using the Rayleigh-Jeans formula, $T_b = I_\nu c^2 / 2k_B \nu^2$, with T_b given by [27]

$$T_b = \frac{(T_s - T_\gamma) \tau}{(1+z)}, \quad \tau = \frac{3c^3 \hbar A_{10} n_H}{16k_B \nu_{21}^2 (H + (1+z) \frac{dv}{dr}) T_s},$$

where n_H is the number density of atomic Hydrogen, A_{10} is the Einstein A coefficient for spontaneous emission, c is the speed of light in vacuum, k_B is Boltzmann's constant and $\hbar = h/2\pi$ with h being Planck's constant. H is the Hubble parameter, r is the comoving distance to redshift z and v is the peculiar velocity along the line of sight. We ignore the contribution of the vector modes to peculiar velocities since it is sub-dominant ($< 1\%$) compared to the scalar mode contribution at scales of interest ($l > 1000$). All quantities except the physical and atomic constants are functions of z . There will be spatial fluctuations in T_b and T_s caused by the fluctuations in n_H and T_g , which in the redshift range of interest are related to the linearly evolved primordial perturbations in standard inflationary cosmology. We will see below that the cosmic strings, if they exist, can add a significant contribution to these fluctuations.

Expanding the fluctuations in the brightness temperature $\delta T_b = T_b - \bar{T}_b$, where \bar{T}_b is the mean brightness temperature, in spherical harmonics, we get the angular power spectrum $C_l(z) = \langle a_{lm} a_{lm}^* \rangle$, where a_{lm} are the coefficients in the spherical harmonic expansion of δT_b . Following [26] we can write,

$$C_l^i(z) = \int \frac{d^3 k}{(2\pi)^3} P^i(k, z) S_l(k, z),$$

where $P(k, z)$ is the baryon (Fourier) power spectrum, index $i = ad$ or cs for adiabatic perturbations from inflation or perturbations from cosmic strings respectively. We have incorporated the 21 cm physics into $S_l(k, z)$ [26].

Results– We calculate $P^{ad}(k, z)$ using CMBFAST [23] and $P^{cs}(k, z)$ using CMBACT [22]. The cosmological parameters are from WMAP3 assuming Λ CDM cosmology [9]. For the cosmic string model in CMBACT we use initial rms velocity of 0.65, wiggleness factor in the radiation era of 1.9 and initial correlation length of 0.13 times the initial conformal time, motivated by numerical simulations [21, 28]. The intercommuting probability of strings is taken to be unity, which means classical strings. The effect of smaller intercommuting probability will translate into a denser network which will make $G\mu$ smaller for the same amplitude of string generated perturbations.

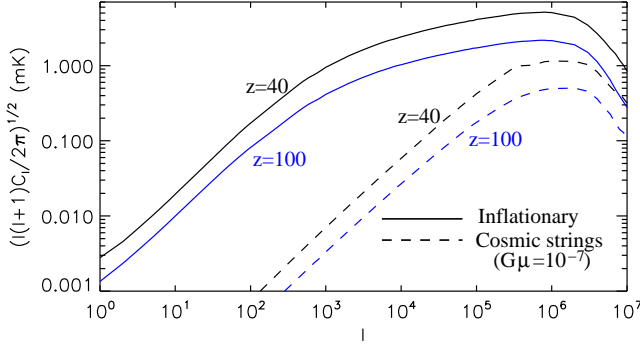


FIG. 1: Angular power spectra from inflationary adiabatic initial conditions (COBE normalized) and cosmic strings.

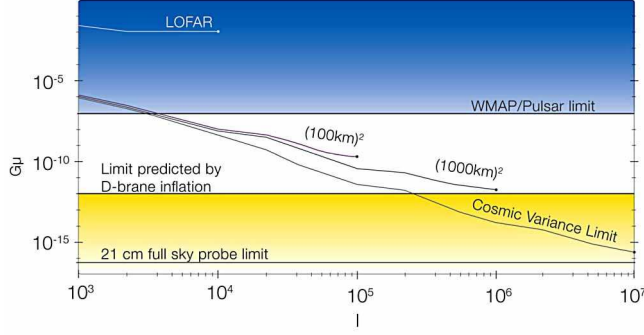


FIG. 2: Constraints from current and future experiments on $G\mu$ for a sky fraction of 10%, bandwidth of 0.4 MHz and integration time of 3 years. Also marked is the constraint on $G\mu$ achievable by a full sky cosmic variance limited experiment for the parameters assumed in the present calculation.

Our predicted constraints on $G\mu$ are therefore conservative.

The angular power spectra for two redshifts are plotted in Fig. 1 for $G\mu = 10^{-7}$ along with the inflationary spectra. One important feature of the cosmic string power spectrum is that it turns over at smaller scales compared to the inflationary adiabatic power spectrum. This is due to the fact that the strings continue to generate perturbations actively at all times. To estimate the constraints possible on $G\mu$ from these observations we calculate the Fisher matrix F_θ for the parameter $\theta = (G\mu/10^{-7})^2$, assuming that all the other parameters are known.

$$F_\theta^i = \sum_z \sum_{l=1}^i \frac{f_{sky}(2l+1)}{2} (C_l^{ad}(z) + C_l^N(z))^{-2} \left(\frac{\partial C_l(z)}{\partial \theta} \right)^2,$$

where $C_l = C_l^{ad} + C_l^{cs}$, f_{sky} is the sky fraction observed and the sum is over all redshift slices and l up to a maximum $l = i$. We use the fact that the bandwidth of any experiment is finite only to estimate the noise C_l^N and the number of redshift slices available and ignore its damping effect on C_l [29]. Bandwidth of 0.4 MHz ensures that the slices are separated by $\gtrsim r/l$ for $l \gtrsim 1000$ and are thus uncorrelated.

The noise in the frequency range $7 < \nu < 47$ MHz is dominated by the sky temperature which follows a power law, $(C_l^N)^{1/2} \propto T_{sky} \propto \nu^{-2.5}$ [30]. For LOFAR [31] we use the noise estimates of [32] at $z=10$, scaled to low frequencies using the above mentioned power law with a bandwidth $\Delta\nu = 0.4$ MHz and integration time of 3 years. These results also apply to LWA [33] which has similar specifications as LOFAR. Fig. 2 shows the constraints on $G\mu$ assuming an error of $1/2(F_\theta^l)^{1/2}$ (the factor of 2 comes in when we convert the error on θ to the error on $\theta^{1/2}$) for LOFAR, two futuristic experiments and the cosmic variance limit assuming that the foregrounds can be removed at required precision [32, 34]. Most of the information on $G\mu$ is at $l > 10^4$ as is clear from an inspection of the power spectra also. For the cosmic variance limit we use the fact that there are more independent modes at high l , $\Delta\nu \approx \nu^2 rH/\nu_{21}lc$ [13] to calculate the number of redshift slices. The curve labeled $(100 \text{ km})^2$ corresponds to a futuristic telescope of size 100 km and collecting area of 10^4 km^2 that will reach out to $l \sim 10^5$ and $G\mu \sim 10^{-10}$. A $(1000 \text{ km})^2$ telescope will be needed to constrain $G\mu \sim 10^{-12}$. Such a telescope might be possible not only on Earth but also in space [35] or on the far side of Moon [36]. To reach $G\mu \sim 10^{-14}$ will require a collecting area of 10^{12} km^2 . Reaching the cosmic variance limit of $\sim 10^{-15}$ may not be possible because the scattering of radio waves by the ionized interstellar medium will limit the smallest angular scales ($\propto \nu^{-2}$) that can be observed [37]. In particular the $l > 10^6$ modes will not be available at all redshifts.

It is clear from Fig. 2 that the information content of the 21 cm signal is huge and can in principle constrain $G\mu \sim 10^{-16}$, if we have a cosmic variance limited measurement on the full sky. This corresponds to a phase transition energy scale of 10^{11} GeV for GUT theories. If we take $\mu \sim 2M_s^2$ for D-brane strings, this means bounds on the superstring mass scale, M_s , down to $\sim 10^{11} \text{ GeV}$ [5]. In reality only a fraction of the sky would be available due to our being confined to the galaxy and small scale modes may not be available because of the interstellar scattering of radio waves, but even then the 21 cm signal has impressive constraining power over $G\mu$. The power at small scales due to cosmic strings is in fact underestimated in this linear calculation. Cosmic strings generate wakes behind them, which have a density contrast of unity, that is not taken into account here and which would enhance the power due to cosmic strings at small scales through non-linear gravitational evolution. This is just the information contained in the power spectrum. Higher order correlations will provide additional discriminating power to check for the signatures of perturbations seeded by a network of cosmic strings. The non-Gaussianities due to cosmic strings would be larger due to the highly non-linear nature of the perturbations at small scales and different from those produced during inflation [14]. This impressive constraint is due the fact that there are large number of modes available at high l so that the statistical errors become very small.

High redshift 21 cm observations thus provide a rare observational window into the superstring theory and supersymmetric grand unified theories through cosmic strings.

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